# Several results concerning the last stages of evolution of close binaries composed of compact companions

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**Abstract**

**Gamma-ray bursts (GRB) are the most powerful transient phenomena in the Universe. Nowadays dozens of speculations on the origin of GRB were undertaken, but so far a single model for the origin of, in particular, short GRBs does not exist. The black hole (BH) - neutron star (NS) coalescence is a promising candidate source for short GRBs. Most of binary mergers numerical simulations were carried out with the purpose of investigating the emission of gravitational waves. Such a scenario consists of an inspiral, merging and ringdown phase. In this paper we present the comparison of the observational results and analytical predictions for a test particle in a quasicircular orbit around the BH. The emission of gravitational waves causes a rapid decrease of the orbital radius and a rise of a *chirp* of radiation. Matter orbiting the black hole would be expected to produce high-frequency oscillations (HFO). Timescales of the coalescence process are of the order of milliseconds and oscillation frequencies of hundreds Hz for a system with a solar mass BH companion. We report on the detection of HFO in two short gamma-ray bursts in this paper. The frequencies and durations of the oscillations are in agreement with the predicted values. A *chirp* phenomenon is identified also. We therefore argue in favor of BH-NS mergers as a scenario for the production of short gamma-ray bursts.**

## 1 Introduction

Gravitational wave (GW) astronomy is an inceptive branch of observational astronomy. There exist many aims for observing with GW detectors (LIGO, VIRGO, TAMA, GEO 600, e.t.c.), e.s. binary black hole - black hole (BH-BH), neutron star - black hole (NS-BH), white dwarf - black hole (WD-BH) systems. There are many evidences for the presence of Supermassive Black Holes in the centers of many galactic bulges. One is the detection of flares from tidally disrupted stars. In the disruption process GW are also emitted. The coalescence of the black hole (BH) with the neutron star (NS) can probably also produce short GRBs with durations of milliseconds.

Most results for the binary mergers obtained from relativistic numerical simulations were carried out with the purpose of investigating the emission of gravitational waves. For such a scenario the inspiral, merging and ringdown phases are typical. As an example we can consider the evolution of a binary system of two nonspinning black holes of equal masses $M_0$, with an initial proper separation of approximately 16.6 $M_0$ (Pretorius 2005). Here we use c = G = 1. The binary is merged within approximately 1 orbit, leaving behind a black hole mass of $M \simeq 1.9\,M_0$. For $M_0 = M_\odot$ the time/space scales are equal to $250\,\mu s$ and $148\,km$, respectively. Approximately 15% of the total scalar field energy does not collapse into black hole. The residual scalar field leaves the vicinity of the orbit quite rapidly, within of the order of the light crossing time of the orbit. About 5 % of the total mass will be emitted as the gravitational waves during the final stages of the collision (ringdown phase) lasting only a few milliseconds.

Faber et al. (2005) have performed relativistic calculations of BH-NS mergers on the assumption that the BH is much more massive than the NS. Adiabatic evolution calculations for neutron stars with low compactness show that the neutron star typically disrupts completely within a few

orbital periods. The majority of the mass transferred onto the black hole is accreted promptly; however a significant fraction ($\simeq 30\%$) of the mass moves outward, some of which will become gravitationally unbound and ejected completely from the system. The remaining portion forms an accretion disk around the black hole, and could provide the energy source for short-duration gamma-ray bursts. As matter accretes onto the BH, it will excite quasinormal ringing modes that could in principle be detected both as gravitational waves and electromagnetic radiation. Gravitational wave emission calculations performed by Faber et al. (2005) showed also the *"chirping"* signal, in which the binary separation decreases while the GW amplitude and frequency increases. This lasts until the onset of mass transfer from the neutron star onto the black hole. At this point we are coming across a much more rapid *"reverse chirp"*, as the gravitational wave amplitude and frequency rapidly decrease while the NS is tidally disrupted.

In recent years SWIFT has detected the afterglows of the short bursts GRB050724, GRB050813, and several others. These bursts together provide evidence for the fact that some short GRBs are occurring due to the merging of NS-NS binary systems (Gehrels & Leonard 2007).

In this paper we present the comparison of the numerical results and analytical predictions based on the detailed calculations for a test particle in a quasicircular orbit rotating about the BH. We report on the detection of high-frequency oscillations during short gamma-ray bursts. These phenomena might be evidence for to the coalescence of a BH-NS binary.

## 2 Background hypotheses

A luminous gamma/X-ray burst can occur when a star passes within the tidal radius of the massive black hole and is disrupted. Disruption begins when the tidal acceleration by the black hole equals the self-gravity of the star. It can be assumed that so far as the Roche lobe lies within the star, mass loss lasts until a star is completely disrupted. The tidal disruption timescale is about of the free-falling time $T_{ff}$. The characteristic time it will take a body to collapse under its own gravity, if no other forces existed to oppose the collapse is

$$T_{ff} = \frac{1}{4}\sqrt{\frac{3\pi}{2G\rho}} \qquad (1)$$

where $\rho$ is the mean density. Note also that the free-falling timescale practically coincides with the oscillation period of a self-gravitating body

$$P \approx \sqrt{\frac{4\pi}{G\rho}} \qquad (2)$$

Numerical calculations of the tidal disruption by Evans & Kochanek (1989) yielded disruption timescale close to above mentioned values also. For the Sun $T_{ff} = 1.78 \cdot 10^3$ s $= 29.7$ min; for a red dwarf of M5 V class $T_{ff} = 685$ s $= 11.8$ min; for a white dwarf and a neutron star of solar mass $T_{ff} = 1.78$ s and $\approx 0.0001 = 0.1$ ms, respectively. Assuming this we can suggest that short-duration gamma-ray bursts, with durations in the milliseconds range, result from the tidal disruption of a neutron star by a black hole.

Thus, a possible scenario for the production of HFO is connected to the coalescence of stellar-mass black holes and neutron stars. Matter orbiting a black hole produces periodic phenomena, which may be identified with HFO. Such coalescence emits both gravitational waves and gamma-rays due to the very hot gas that arises as a result of the tidal disruption of a neutron star. The emission of gravitational waves causes a rapid decrease of the orbital radius and produces a *chirp* of radiation (Shutz 2001). The details of the GW waveforms (and luminosity) depend on geometrical characteristics, masses and orientation of the binary. They also show striking dependence on the stiffness of the equation of state of the neutron matter which has been shown in numerical simulations (Lee 2000, Rosswog 2004). Depending on the polytropic index the radiation signal may reveal periodic peaks (HFO) or drop abruptly to zero after the star is disrupted.

It appears that HFO contain encoded data about both the geometry and physics of the BH-NS binary. Of interest is an important feature of a Reissner-Nordström black hole, namely the possibility to convert the reflected gravitational waves arising from the coalescence process to electromagnetic radiation (Chandrasekhar 1983). The predicted rate from such a conversion may amount to a few tens of percents at resonant frequencies that depend on the BH size (Gunter 1981).

The gravitational-wave luminosity of two stars of equal mass $M$ in an orbit of radius $R$ is in order of magnitude (Shutz 2001)

$$L \simeq \frac{1}{80}\frac{c^5}{G}\left(\frac{GM}{Rc^2}\right)^5 \qquad (3)$$

It is possible to calculate that close BH-NS binaries with $M = M_\odot$ and $R < 1/6\, R_\odot$ can radiate more energy in gravitational waves than the Sun in light. Then note that the gravitational wave amplitude for a binary system is

$$h_{binary} \simeq \frac{1}{2} \frac{GM}{r\,c^2} \frac{GM}{R\,c^2} = \frac{1}{8} \frac{r_g^2}{r\,R} \qquad (4)$$

Perturbations of the metric in the vicinity of coalescing stars ($r \sim R \sim r_g$, where $r_g = 2GM/c^2$ is the gravitational radius), may be extremely large ($h \simeq 1$). An important point is that the variations of gamma-ray intensity will carry a print of strong gravitational fields from in-spiraling orbits (gravitational waveforms) on them. It may provide another insight into mergers of stars and BHs. These events are expected to be detected by future space-based gravitational-wave interferometers. Remarkably, similar results can be obtained, analyzing gamma-ray HFO.

Thus, the most simple model suggests that the frequency of HFO would be equal to the local Keplerian frequency $f = (GM/R^3)^{1/2}/2\pi$. Detailed calculations (Chandrasekhar 1983; Novikov & Frolov 1986)have shown, that for a test particle of mass $m$ the stable circular orbits around the BH of mass $M$ exist for $R > 3\,r_g$ only. The binding energy of the test particle in close proximity to the critical orbit is $E \simeq 0.06\,mc^2$. During each cycle in the critical orbit the particle radiates the gravitational energy $\Delta E \approx 0.1\,mc^2(m/M)$. In general, inside the critical orbit the particle moves in a spiral curve with $N$ cycles before coalescence, where $N \approx (M/m)^{1/3}$. Gravitational wave radiation causes the orbital radius to shrink, thus for $R \gg r_g$ it has the following form (Landau & Lifshitz 1973)

$$\frac{dR}{dt} = -\frac{8}{5}\,c \left(\frac{m}{M}\right)\left(\frac{r_g}{R}\right)^3 \qquad (5)$$

If the initial radius of the Newtonian orbit $R$ of two stars of the equal mass $M_\odot$ ranges from 4 to 10 gravitational radii, i.e. approximately 12 to 30 km, the frequency of HFO will lay between 350 Hz and 2.2 kHz. In this case a close binary appears to be extremely unstable. The gravitational-wave energy released around the critical orbit can be as high as $10^{53}$ ergs. Integrating the differential equation (5) in time we will get the survival time of a binary system with initial conditions mentioned above. This time is around 0.015 s. Strikingly, the gamma-ray bursts considered here demonstrate both the HFO frequencies and the times of oscillations close to the mentioned values.

Note that all the above estimates should be treated as approximate results. These are valid for test particles orbiting black holes. Coalescence of the BH-NS binaries requires the solution of the set of Einstein equations. To obtain the right expression for the orbital frequency, a calculation in general relativity is necessary. The case of a circular orbit $R = 3\,r_g$ corresponds to a singular trajectory. It has no analogue in Newtonian theory. Detailed relativistic calculations show that the local Keplerian period $T_K$ and the rotational period $T$ for an outside observer are related as (Chandrasekhar 1983)

$$T_K = \left(4\pi^2 R^3/GM\right)^{1/2} \qquad (6)$$

$$T = T_K(1 - 6\,\mu)^{-1/2} = \pi \frac{r_g}{c}[\mu^3(1 - 6\,\mu)]^{-1/2} \qquad (7)$$

where $\mu = r_g/2R$. It can be shown, that while an orbital radius $R$ is larger than $4\,r_g$ the observed frequency will be *chirping up* (see Fig. 1). For $R < 4\,r_g$ it goes down ("*reverse chirp*"). This can be explained by the time dilation close to the event horizon of the black hole. The observed period $T$ diverges to infinity when $R$ approaches $3\,r_g$. Thus, the chirping timescale determines both the binary mass and size.

Lee & Ramirez-Ruiz (2002) studied numerically the hydrodynamic evolution of massive accretion disks formed as a result of the disruption of a neutron star by a black hole in the context of gamma-ray bursts formation. A gas mass $\simeq 0.1 - 0.25\,M_\odot$ survives in the orbiting debris, which enables strong magnetic fields $\simeq 10^{16}$ G to be anchored in the dense matter long enough to power short duration GRBs. They estimate the energy released from the system through magnetic-dominated mechanisms and find it can be as high as $10^{51}$ ergs during an estimated accretion timescale of 0.1...0.2 s.

Impact of two stars may drive their own oscillations. The amplitudes of the forced oscillations may be large enough to induce a variety of nonlinear effects, in particular, generation of both higher and fractional harmonics, etc.

HFO may also be related to the fundamental pulsation frequency of coalescing stars. The oscillation frequency of a self-gravitating body is $f \sim (G\bar{\rho}/4\pi)^{1/2}$ (Shutz 2001). This quantity is of the same order as the Keplerian orbital frequency. For BH-NS binaries with $M = M_\odot$ and $R_{NS} \simeq 10$ km, $R_{BH} = r_g = 2GM_\odot/c^2 = 2.95$ km, the fundamental pulsation frequency is 1.6 and 9.9 kHz, respectively. In such a case HFO can be regarded as a probe for the structure and the equation of state of neutron stars as well as an asteroseismology probe for black holes.
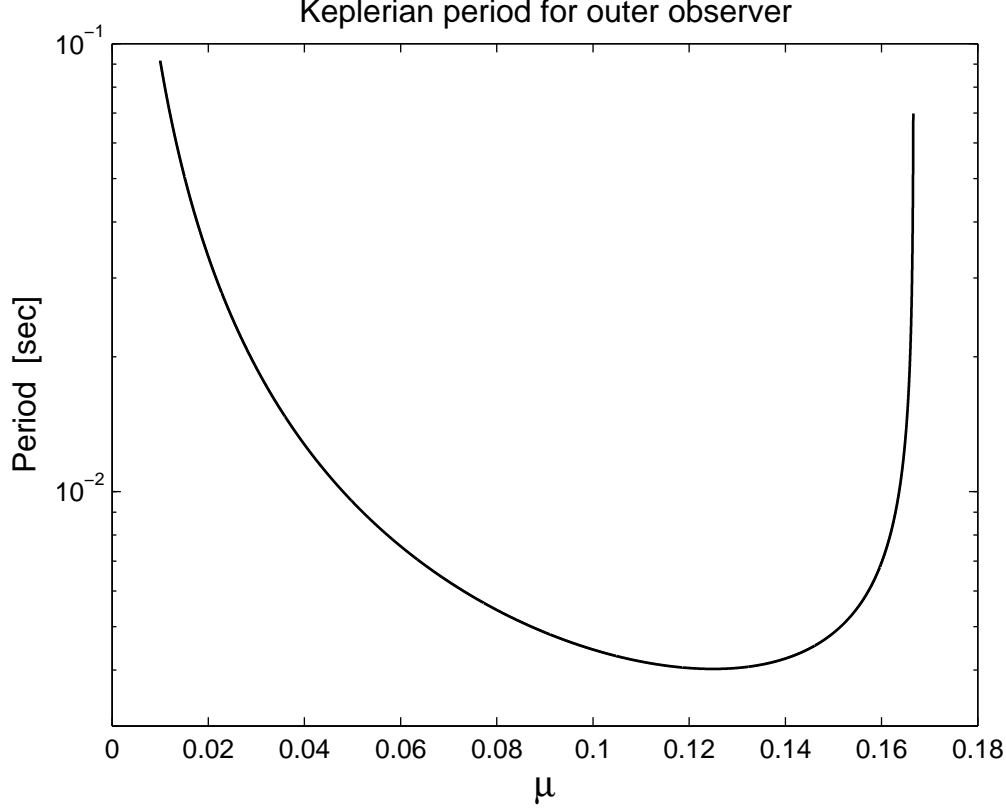
Figure 1: The variation of the Keplerian rotational period for an external observer as derived from equation (7). The abscissa units are $\mu = r_g/2R$ for $M = M_\odot$. The scale of the ordinate is given in seconds.

The radiation from a relativistic object is a nontrivial problem, which requires treating it relativistically using Lorentz transform. The Lorentz factor $\gamma$ appears in Special Relativity. It is defined as:

$$\gamma \equiv \frac{1}{\sqrt{1 - \beta^2}} \; ; \; \beta = \frac{v}{c} \qquad (8)$$

where $c$ is the speed of light and $v$ is the velocity of the object. An object moving with respect to an observer will seem to radiate in a different way because of the time dilation and length contraction. Intensity for external observer is (Lightman et al. 1975)

$$L_{observ} = \frac{L}{4\pi R^2} \frac{1}{\gamma^4(1 + \frac{v}{c} cos(\theta))^4} \qquad (9)$$

where $L$ is the luminosity and $\theta$ is the angle between the line of sight from object to observer and the direction of motion. We can see that intensity $L_{observ}$ depends both on the Lorentz factor $\gamma$ and the angle $\theta$. When $\beta$ is much less than unity, the radiation pattern is symmetric both in the forward and backward directions. However, as $\beta = v/c \to 1$, the radiation pattern becomes more and more concentrated to the forward direction. For a highly relativistic object, the radiation is emitted in a narrow cone which axis is aligned to the direction of motion.

HFO in GRBs may be treated as a result of intensity modulation caused by the variation in an angle $\theta$ at a relativistic rates of movement in a close binary system. Modulation depth may be defined as the peak value of intensity divided by its minimum value. We define also a beamwidth of varying intensity $L_{observ}$ as $FWHM$ (full width at half maximum). Both the modulation depth and beamwidth of the HFO signal are shown in Fig. 2 as function of $\beta$ following equation (7).

So if $\beta = 0.5$, the HFO intensity changes by approximately two orders of magnitude, and the half-power beamwidth is equal to about $30°$. In that case we can observe the HFO phenomenon if the inclination angle to the line of sight of a emitting system is more than $60°$.

Not all short bursts have identical features. Since at relativistic velocities the radiation is emitted in a narrow cone, some bursts will not show the HFO effect because of geometrical reasons.
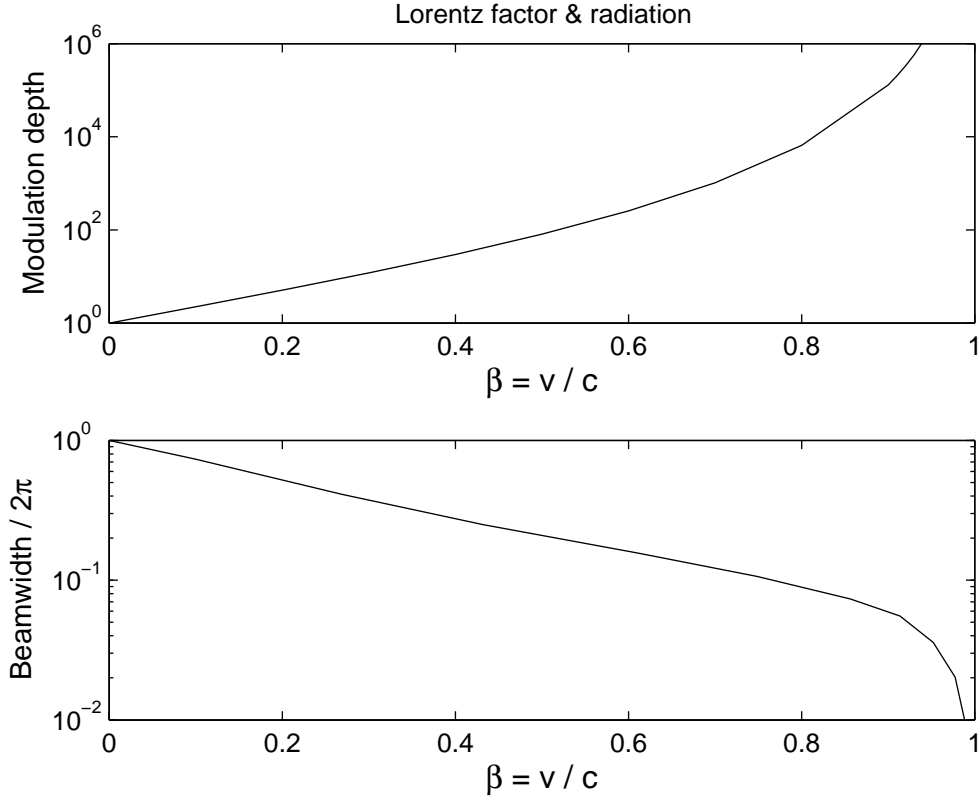
Figure 2: Variations in both modulation depth and half-power beamwidth of the HFO signal as function of $\beta$ as follows from equation (7).

# 3   Individual GRB events: BATSE triggers # 432 and 512

We have performed searches for high-frequency periodicities in two short gamma-ray bursts on millisecond timescales. In our analysis we used the TTE data from the BATSE 3B catalogue (Meegan et. al. 1996). The TTE data contains the arrival time over a 2 microseconds time bin, the energy of each photon and the number of the detector.

For trigger number 423, Cline et al. (1999) give the $T_{90}$ burst duration followed from TTE fit equal to $0.050 \pm 0.002$ s. They also note the detailed structure of BATSE trigger 512, which has the finest time structure of any GRB observed to date - possibly down to the 20 $\mu s$ level. This is a very bright burst with maximum power in the third energy channel (100-300 keV). Its duration is $0.014 \pm 0.0006$ s only.

We investigate the intensity oscillations in short gamma-ray bursts with wavelet analysis (Torrence & Compo 1998).

Fig. 3 shows the light curve of BATSE trigger 432 in the energy channel 50-100 keV and its wavelet power spectrum. The white contours give the significance levels of 90%, 95% and 99%, respectively. The dark areas point to the burst oscillations with a significance level of $> 99.9\%$. This spectrum reveals a significant peak around 5 ms.

The plot shows also the *chirping* signal with an oscillation period varying in time. The oscillation was detected at the burst onset at a frequency of about 300 Hz, and it increased in frequency over the following 30 ms of the burst rising to a maximum of about 1000 Hz. From that moment and forth the observed frequency is *chirping* down. It is decreasing over the following 20 ms of the burst decay to a minimum of about 400 Hz. As was mentioned above, its decrease may be explained by the time dilation close to the event horizon of a black hole. The bead-like structure of the spectrum depends, in particular, on the unstable behavior of the high-frequency components. This *chirping* phenomenon points to a coalescing binary with a black hole companion of stellar mass.
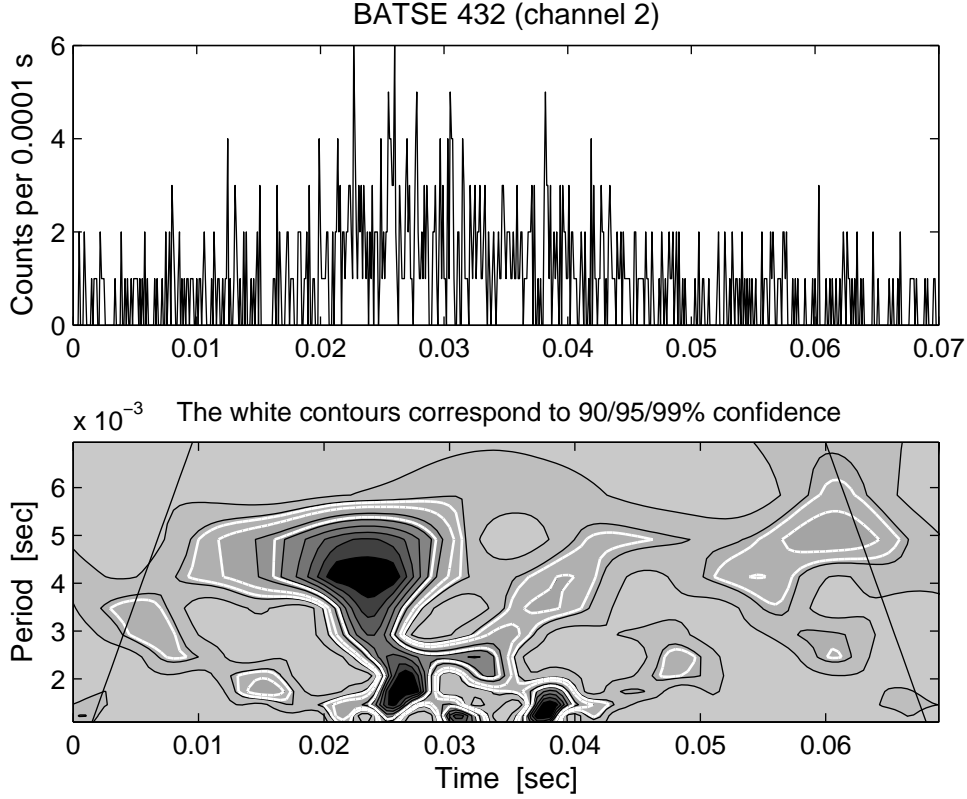
Figure 3: The light curve of BATSE trigger 432 in the energy channel 50-100 keV from the TTE data binned to 100 $\mu s$ resolution (top) and its wavelet power spectrum (bottom).

The wavelet transform allows us to extract the *chirping* signal from the instrumental noise. Equation (5) gives the orbital shrinking time of 0.015 s for the BH of the solar mass. This is in agreement with an estimate of the burst duration from a fit to the TTE light curve. The minimum period of the orbit in the range $(1.5\ldots3)\,10^{-3}$ s provides also the mass of BH $\simeq M_\odot$ according to equation (7). As discussed above in close proximity to the critical orbit we can probably observe only a few cycles of radiation before coalescence. For an orbital radius $R < 4\,r_g$ the observed frequency was found to be *chirping* down.

Fig. 4 shows at least two curves *chirping* down in the local wavelet power spectrum of BATSE trigger 512. As noted by Lee & Ramirez-Ruiz (2002), a gas mass $0.1\ldots0.25\,M_\odot$ survives in the orbiting debris after the neutron star is disrupted by the black hole. Multiple chirping curves might point to fragments of different mass falling into th BH. Note that equation (6) for local Keplerian period $T_k$ is valid only for the infinitesimal mass particles orbiting black holes.

Fig. 5 also shows the decomposed light curves of BATSE trigger 512, following the approach offered by Torrence & Compo (1998). The thin curve represents the high-frequency range, 0.2 ms < period < 1 ms. The thick curve encompasses the low-frequency diapason with periods > 1 ms. When added, they would form the original light curve. The dashed lines mark the $\pm$ 3 sigma error corridor. Our results referring to reconstruction can be summarized as follows.

- The oscillations occur only during the burst phase. They appear suddenly with a rise time comparable with resolving time of 100 microseconds. Recall that the dynamical time of the disruption of a NS as mentioned above is about 0.1 ms.

- Of interest for theory are oscillations that are visible in the both low- and high-frequency light curves. The HFO amplitude can be large, a significant fraction of the energy released is radiated in HFO. The contribution of HFO to the total luminosity of the burst event is estimated to be roughly up to 50%.
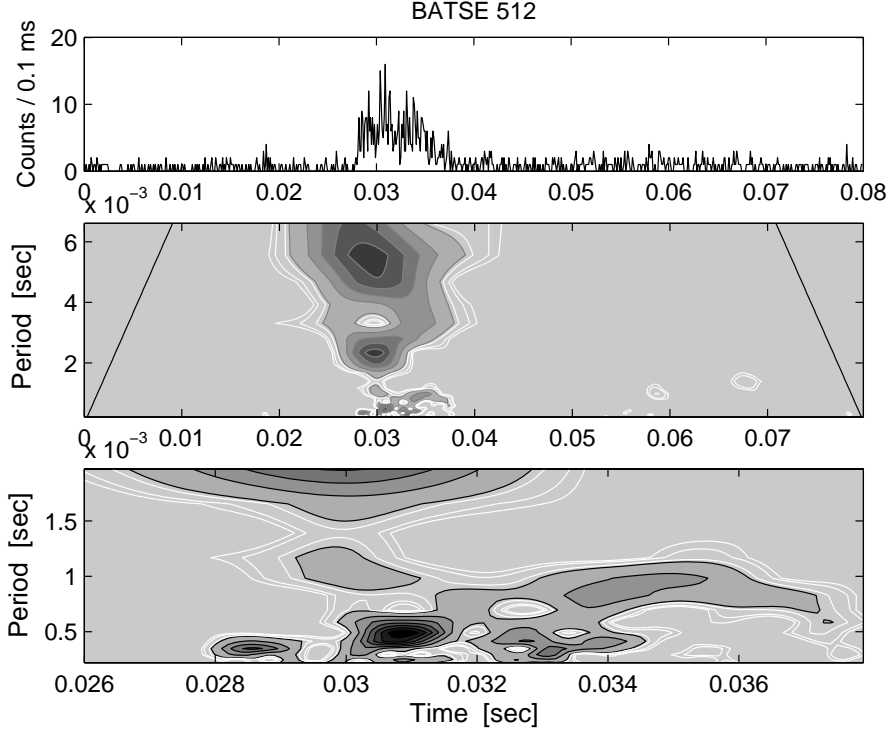
Figure 4: BATSE trigger 512. The upper panel shows the light curve in the energy channel 100-300 keV with a sampling time of 100 $\mu s$. The middle panel shows the local wavelet power spectrum. The lower panel with a magnified portion of the high-frequency spectrum demonstrates a *chirp* phenomenon. The outside white contours enclose regions of power of greater than 95% confidence relative to white noise.

- The instantaneous frequency of HFO varies with time, as may be proved from the peak-to-peak time measurements in Fig. 5. It appears that the HFO amplitude changes periodically. The amplitude of modulation of oscillations amounts up to 100%. As one might expect from equation (9) and Fig. 2 a radiating object moves with a velocity $\geq 0.6\,c$.

- The low-frequency light curve shows a few cycles with a mean period between 1.9 ms and 4.4 ms, chirping down. The amplitude of modulation of low-frequency oscillations amounts only to 30%. From here a radiating object moves with the velocity $\simeq 0.2\,c$.

- It seems, we deal with fragments of matter moving in orbits of different hight.

## 4   Conclusion

The detection of high-frequency oscillations in short gamma-ray bursts provides a new prospect for exploring the nature of this mysterious phenomenon. HFO with periods in the range of milliseconds and magnitudes of a few tens of percent of the burst luminosity may be related to the accretion of debris formed after the tidal disruption of a neutron star by a black hole in a coalescing binary system. For such a scenario, one would expect a few cycles of radiation at an orbital frequency *chirping up and down*. This phenomenon indeed is seen in the bursts considered here.
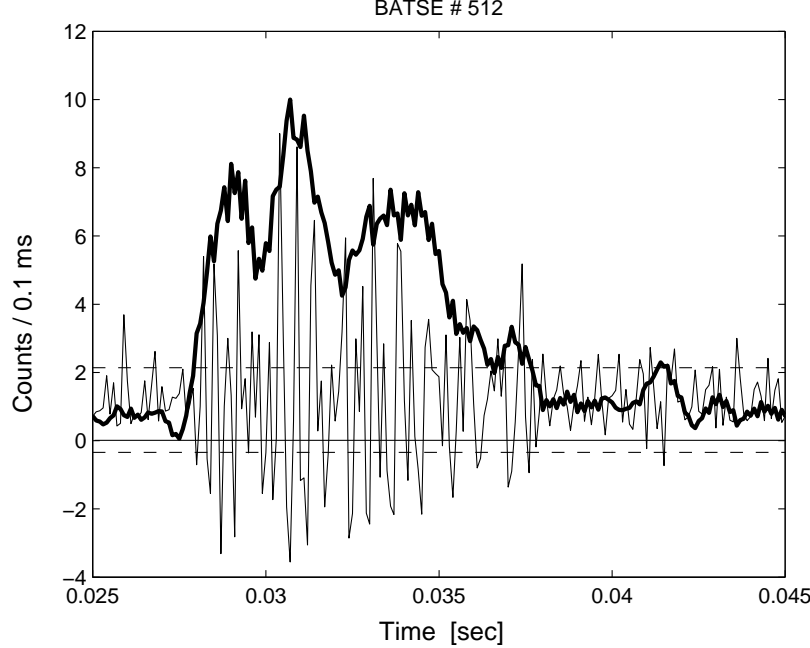
Figure 5: The decomposed light curve of BATSE trigger 512 (see text). The data of the energy channel 100-300 keV is used with a resolving time of 100 $\mu s$ in two frequency bands. The thin curve represents the high-frequency range (0.2 ms < period < 1 ms) with the $\pm$ 3 sigma error corridor (the dashed lines). The thick curve encompasses the low-frequency diapason with periods > 1 ms.

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